NEW ASPECTS OF ABELIAN CONFINEMENT

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cartoon of a baryon in SU(3) QCD(adj)
Outline

1 continue Tin Sulejmanpasic’s talk from this morning, focusing on N-ality properties of strings

2 remarks on global structure and a “baby-S-duality” in deformed YM
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2. remarks on global structure and a “baby-S-duality” in deformed YM

This talk is on calculable confinement in deformed-YM and QCD(adj)/SYM on $\mathbb{R}^{1,2} \times S^1$ with small $S^1$ - not the real world!

Feel the need for some
“Philosophical” remarks
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ii. One might hope that upon studying a solvable example, new unexpected and interesting features of more general utility will be encountered.
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iii. Once an analytical approach is understood within its region of validity, it is tempting to push it to, and even beyond, its limits—i.e. the approach might contain qualitative lessons for phenomenological models of the real strongly coupled system.
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talks by Tin Sulejmanpasic Edward Shuryak Eric Zhitnitsky E.P. Aleksey Cherman Gerald Dunne Mithat Unsal Gokce Basar are - directly or indirectly, to a lesser or larger extent - inspired by these small-L studies

my own involvement began here, during CAQCD 2008!
Motivation

Confinement is ubiquitous, ‘old news’. However, it is quantitatively understood - within continuum QFT, starting from the microscopic QFT degrees of freedom and in a controlled manner - only in a few cases.

- Seiberg-Witten theory: N=2 super YM with N=1 soft mass, abelian confinement Douglas Shenker, Hanany Strassler Zaffaroni,...

- monopole confinement in abelian Higgs model and in related (dual) models with nonabelian strings Gorsky, Shifman, Yung...

- confinement on $R^3 \times S^1$, abelian Unsal, Shifman, Yaffe...

lattice - numerical experiment - confining flux tubes exist, for sure - but not (...) given gauge invariant insight into dynamical mechanism

string theory - under control in regimes quite far from asymptotically free QFT

So, it is of some interest to better study the few understood QFT cases and their relations to each other and to lattice.
We study SU(N) in the regime $\frac{NLA}{L} \ll 1$

**QCD(adj):** YM with $n_f$ adjoint Weyl fermions; $n_f = 1$ is SYM

**dYM:** pure YM with particular double-trace “deformation” or adjoint fermions of mass $\sim O(1)/(NL)$

**key features:**

1. Dynamical abelianization $SU(N) \rightarrow U(1)^{N-1}$
2. At distances $>> NL$ weak coupling
3. Relevant d.o.f. are $N-1$ dual photons
4. Mass gap for dual photons due to proliferation, or “condensation”, of
   - magnetic bions - QCD(adj)/SYM
   - monopole-instantons - dYM
to continue Tin Sulejmanpasic’s talk begin with
- magnetic bions - QCD(adj)/SYM with SU(3) gauge group:
two dual photons - contour plot of bion-induced potential
(picture in lieu of formulae)
magnetic bions - QCD(adj)/SYM with SU(3) gauge group:  
two dual photons - contour plot of bion-induced potential

dual photon plane

periodicities:
w1, w2: weight vectors of SU(3)

center symmetry:
acts on the dual photons
as a Z_3 subgroup of the Weyl group
\( \mathbf{\sigma} \rightarrow P \mathbf{\sigma} = 120 \) degree rotation

coordinate free for SU(N): product of Weyl reflections wrt simple roots
\[
P = s_{\alpha_{N-1}} s_{\alpha_{N-2}} \cdots s_{\alpha_2} s_{\alpha_1} \quad s_\alpha \mathbf{v} = \mathbf{v} - 2\mathbf{\alpha} \frac{\mathbf{v} \cdot \mathbf{\alpha}}{\mathbf{\alpha} \cdot \mathbf{\alpha}}
\]
magnetic bions - QCD(adj)/SYM with SU(3) gauge group: two dual photons - contour plot of bion-induced potential
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periodicities:
w1, w2: weight vectors of SU(3)
3 vacua - 1,2,3
broken discrete chiral symmetry
(preserve center symmetry
120 degree rotn + w_k shift)
- magnetic bions - QCD(adj)/SYM with SU(3) gauge group: two dual photons - contour plot of bion-induced potential

- monodromy of dual photon around Wilson loop

**Confining strings?**

\[ \oint_C d\sigma = 2\pi\lambda. \]

**3 vacua - 1,2,3**

**broken discrete chiral symmetry**

**periodicities:**

- w1, w2: weight vectors of SU(3)

**dual photon plane**

- weight lattice, but are restricted by the condition that operators in faithful representations of take values in the group lattice long-distance abelian theory, as evident from the final expressions (is essentially the electric field operator).

- We already discussed that the electric weights \( \hat{\nu} \), \( \hat{\nu} \) are single valued around all allowed charges, dynamical or probes, in a gauge theory with gauge group called the magnetic or dual center symmetry. This symmetry, being generated by shifts of \( Z_G \) and \( Z_m \) are electric and magnetic weights (see below) and

- and on the Wilson, 't Hooft and dyonic operators, that for 

- To motivate the expressions that follow, we note that our long-distance theory is abelian,
- magnetic bions - QCD(adj)/SYM with SU(3) gauge group:
  in semiclassical regime extremize classical action with monodromy around Wilson loop:

\[ e^{\frac{i}{2} \oint_C A^{(3)}} \]

\[ \oint_C d\sigma = 2\pi \lambda \]

monodromy

for, e.g., a weight of the fundamental, say \( w_1 \)
- magnetic bions - QCD(adj)/SYM with SU(3) gauge group: in semiclassical regime extremize classical action with monodromy around Wilson loop:

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Monodromy

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magnetic bions - QCD(adj)/SYM with SU(3) gauge group:
in semiclassical regime extremize classical action with monodromy around Wilson loop:

\[
e^{i \frac{1}{2} \oint_{C} A^{(3)}}
\]

\[
\oint_{C} d\sigma = 2\pi \lambda.
\]
monodromy

now, w1-string in vacuum 1

for, e.g., a weight of the fundamental, say w 1
QCD(adj)/SYM with SU(3) gauge group:
- the story told already in Tin Sulejmanpasic’s talk, about “strings from flesh and blood” ending on domain walls also holds, clearly...
- quarks with charges in all weights of the fundamental have the same string tension, by the Z_N center symmetry

- for higher N-alities quarks string tensions are the same in every Z_N Weyl orbit [su(4)ex.] (no free lunch: no complete “N-ality only” dependence in abelian regime)
- very different from other theories with abelian confinement, notably Seiberg-Witten theory (SW)
- “baryon vertices” exist, as opposed to linear-only baryons in SW (show later)
magnetic bions - QCD(adj)/SYM vs Seiberg-Witten theory

qualitative difference is because:

1. in SW there are N-1 condensing objects, in QCD(adj)/dYM there are N “condensing” monopole instantons: the extra “KK monopole” makes all the difference

2. in SW Weyl group totally broken, in QCD(adj)/dYM a $\mathbb{Z}_N$ subgroup exact, (in gauge invariant terms, the $\mathbb{Z}_N$ Weyl is really the action of the center)

Composite nature of strings in QCD(adj)/SYM is, ultimately, due to “composite” nature of condensing objects:

MM* “molecules” rather than M’s themselves.
magnetic bions - QCD(adj)/SYM vs Seiberg-Witten theory

in pictures

recall Seiberg-Witten theory:

- two types of monopoles (SU(3)) with (co) root-lattice charges, label k=1,2,...N-1
- ANO vortices are strings confining quarks in the highest weight k-index antisymmetric representation

mesons

ANO vortex of unit N-ality
ANO vortex of N-ality 2

- nondegenerate mesons

(k-th component of fundamental bound by k-string and an anti k-1-string)

only linear baryons
(more dramatic for N>3)
Seiberg-Witten vs QCD(adj)/SYM

in pictures

**Seiberg-Witten theory:**

mesons

- \( \nu_1 \rightarrow \nu_1 \)
- \( \nu_2 \rightarrow \rightarrow \nu_2 \)
- \( \nu_3 \rightarrow \rightarrow \nu_3 \)

**QCD(adj)/SYM:**

- \( \nu_1 \rightarrow \nu_1 \)
- \( \nu_2 \rightarrow \nu_2 \)
- \( \nu_3 \rightarrow \nu_3 \)

"baryon vertex"

only linear baryons (more dramatic for N>3)
Seiberg-Witten vs QCD(adj)/SYM vs dYM

N-1 monopoles condense
N bions condense (non-composite)
N monopoles condense so non-composite strings

Seiberg-Witten theory:

mesons

QCD(adj)/SYM:

\[ (1+P+P^2)w_1 = 0 \]

Seiberg-Witten theory:

N monopoles condense so non-composite strings

“baryon vertex” (DW junction)

only linear baryons (more dramatic for N>3)
Finally, a brief mention of an “application” and a curiosity...

The picture of strings and DWs in dYM and QCD(adj) can be used to elucidate the distinct global structure - discrete theta angles “p” Aharony/Seiberg/Tachikawa, Kapustin/Seiberg -of $[SU(N)/Z_k]_p$ theories in a rather pedestrian physical manner.

As an application, the low-T/high-T Kramers-Wannier-like duality near $T_c$ on $R^2 \times S^1 \times S^1$ [Simic, Unsal; Anber, Unsal EP] can be shown to be consistent with global structure and $(ST)^3 = 1$, eliminating some puzzles:
dYM on \( \mathbb{R}^2 \times S^1 \beta \times S^1 \_L \)

monopole-instanton “events” (world lines in smaller L-circle, not shown)

static W-bosons’ worldlines

electric-magnetic “charge” system:
- very dilute at \( T \ll 1/L \), typical distances \( \gg 1/T \), so \( \sim 2d \)
- 2d thermal deconfinement transition at \( T \sim g^2/L \)
  ~ to 3d Polyakov model Dunne, Kogan, Kovner, Tekin; Lecheminant, Gogolin, Nersesyan

- here: focus on symmetries and duality:
dYM on 
$$\mathbb{R}^2 \times S^1_\beta \times S^1_L$$

2d electric-magnetic Coulomb gas partition function invariant under

$$\hat{S} : (y_m, y_e) \rightarrow (y_e, y_m) , \quad (q^e \alpha_i, q^m \alpha_i^*) \rightarrow (q^m \alpha_i^*, -q^e \alpha_i) , \quad \frac{g^2}{4\pi LT} \rightarrow \frac{4\pi LT}{g^2}$$

exchanges low and high-T, electric and magnetic charges

high-T $Z_N$ center broken

$$\langle W(r)\tilde{W}(0)\rangle \bigg|_{r \to \infty} = \begin{cases} 1, & \text{thus } \langle W \rangle = \pm 1 \text{ for } T > T_c = \frac{g^2}{4\pi L} \\ e^{-\frac{\sigma \pi}{T}}, & \text{thus } \langle W \rangle = 0 \text{ for } T < T_c. \end{cases}$$

low-T $\tilde{Z}_N$ center broken

$$\langle H(r)\tilde{H}(0)\rangle \bigg|_{r \to \infty} = \begin{cases} e^{-\frac{\sigma \pi}{T}}, & \text{thus } \langle H \rangle = 0 \text{ for } T > T_c = \frac{g^2}{4\pi L} \\ 1, & \text{thus } \langle H \rangle = \pm 1 \text{ for } T < T_c. \end{cases}$$

so, can’t be a self-duality, i.e. high T of SU(2) to low T of SU(2)

- same as Seiberg et al puzzle - from Ising to N=4 S-duality... all done “wrong” so far!
dYM on $\mathbb{R}^2 \times S^1_\beta \times S^1_L$ resolution:

$S$ action $\sim$ to that of S-duality in N=4 SYM, consistent with $(ST)^3 = 1$

- different global structures
- theories are seen in, say, the XY’ model description of the 2d Coulomb gas (in complete analogy with YM)

- only example of S-duality in non-susy YM (I know of)
- emergent S only, for sure (curiosity only?)

\[
\begin{align*}
su(3): & \quad T \subset SU(3) \xrightarrow{S} (SU(3)/\mathbb{Z}_3)_0 \xrightarrow{T} (SU(3)/\mathbb{Z}_3)_1 \\
su(4): & \quad \bigcup_T (SU(4)/\mathbb{Z}_4)_0 \xrightarrow{S} (SU(4)/\mathbb{Z}_4)_1 \xrightarrow{T} (SU(4)/\mathbb{Z}_4)_2 \xrightarrow{S} (SU(4)/\mathbb{Z}_4)_3 \xrightarrow{S,T} (SU(4)/\mathbb{Z}_2)_+ \end{align*}
\]
Conclusions/Questions

Abelian confinement can be quite rich and diverse.

On $\mathbb{R}^3 \times S^1$, at small-$L$, due to
1.) center symmetry and
2.) the composite nature of “condensing” magnetic objects
its properties are quite distinct from other theories with abelian
confinement (e.g. Seiberg-Witten).

What happens as $L$ becomes large? The small-$L$ theory can be
connected to both pure YM and to softly-broken (to $N=1$)
Seiberg-Witten theory in this way. How do the various kinds of
strings we discussed evolve into each other?

It has been conjectured that upon transition to the nonabelian regime, the
magnetic flux gets collimated into ‘center vortex sheets’ percolating through
spacetime and disordering Wilson loops… lattice evidence [Greensite et al; deForcrand, D’Elia]
If so, it would appear that global structure plays a role in the dynamics in the
nonabelian regime, while it does not in the abelian (both on $\mathbb{R}^3 \times S^1$ and in SW).